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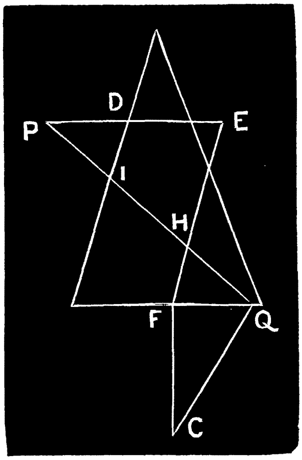


For $\dfrac{\triangle FHQ}{\triangle PHE}=\dfrac{FQ^2}{PE^2}=\dfrac{PE^2-PD^2}{PE^2}=1-\dfrac{PD^2}{PE^2}$ $=1-\dfrac{\triangle PDI}{\triangle PEH}$;

$\therefore \triangle FHQ=\triangle PEH-\triangle PDI=DIHE$; $\therefore \triangle FHQ+IHFA=DIHE+IHFA=DEFA$, or $AIQ=DEFA$.

Also solved by L. E. Newcomb, Los Gatos, California.

219. Proposed by L. E. DICKSON, Ph. D., The University of Chicago.

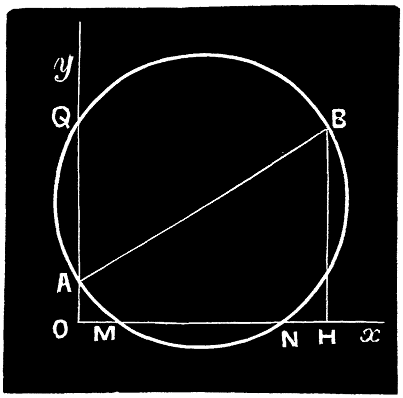
Devise a simple geometric solution of the general quadratic equation.

I. Remark by W. W. LANDIS.

A solution may be found in Klein's *Vorträge über ausgewahlte Fragen der Elementargeometrie*, pp. 28-31; in Beman and Smith's translation, p. 34.

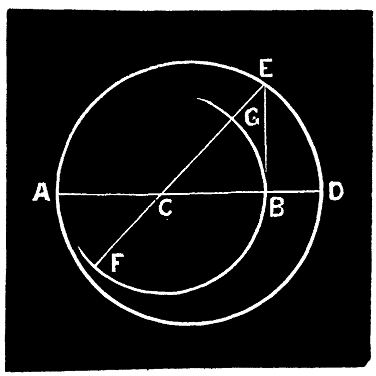
II. Solution reported by the PROPOSER.

The elegant solution by Lill (reported without proof by d'Ocagne at the Second International Congress of Mathematicians, Paris, 1900) is so simple that the Proposer has used it in his courses in elementary algebra. For the graphic solution of $x^2+px+q=0$, choose two perpendicular lines Ox and Oy, lay off unit length OA on Oy, length OH on Ox containing $-p$ units (to right or left of O, according as $-p$ is + or $-$), length HB on parallel to Oy containing q units. If the circle on AB as diameter cuts Ox at M and N, then OM and ON, on the same scale, are the required roots. In proof, let Q be the second point of intersection of the circle Oy, then $OQ=HB$, since $OHBQ$ is a rectangle; $OM=NH$ by eqnality of triangles OQM and HBN. Hence $OM.ON=OA.OQ=q$, $OM+ON=OH=-p$.



III. Solution by B. F. FINKEL, A. M., M. Sc., 204 St. Marks Square, Philadelphia, Pa.

Let $ax^2+bx+c=0$, be the general quadratic. On the line AD, lay off $AB=2$ units, and $BD=c/2a$. On AD as a diameter describe the circle AED. At B erect the perpendicular BE. With E as a center and a radius equal to $b/2a$, describe an arc intersecting AD, or AD produced, in C. Then with C as a center and a radius equal to CB describe the circle FBG intersecting EC in G and F in the order



E, G, C, F. Then EG is equal to the value of one root of the quadratic and EF is equal to the other. For

$$BC=\sqrt{CE^2-BE^2}=\sqrt{CE^2-AB.BD}=\sqrt{\frac{b^2}{4a^2}-\frac{c}{a}}=\frac{1}{2a}\sqrt{b^2-4ac}=CG=CF.$$

Then $EG=\dfrac{-b+\sqrt{(b^2-4ac)}}{2a}$ and $EF=\dfrac{-b-\sqrt{(b^2-4ac)}}{2a}$.

This construction only gives the absolute values of the real roots. The corresresponding algebraic values must be assigned.

If $b^2/4a^2=c/a$, then $CE=BE$, which equals x.

If $b^2/4a^2<c/a$, the construction is impossible.

IV. Solution by A. H. HOLMES, Brunswick, Maine.

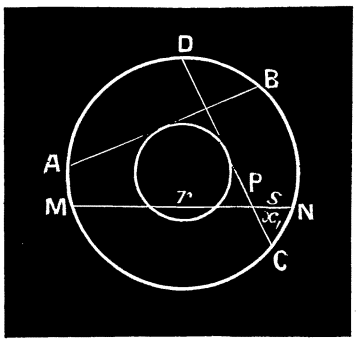
Describe a circle of radius a, and from a point on the circumference A draw tangent $AB\sqrt{b}$. Then from B draw through the center of the circle O the line BD cutting the circumference in C and D. By the principles of plane geometry the lines CB and BD will represent the unknown quantity in the quadratic equation $x^2+2ax=b$.*

V. Solution by L. LELAND LOCKE, Brooklyn, N. Y.

We have four cases to consider. These may be reduced to two.

$$\text{I.}\quad \begin{cases} x^2-px+q=0 \ldots\ldots\ldots\ldots (1), \\ x^2+px+q=0 \ldots\ldots\ldots\ldots (2); \end{cases}$$

$$\text{II.}\quad \begin{cases} x^2+px-q=0 \ldots\ldots\ldots\ldots (3), \\ x^2-px-q=0 \ldots\ldots\ldots\ldots (4). \end{cases}$$



Let $q=r.s$, r and s being any suitable factors of q.

Case I. Equations 1, 2. Draw a circumference 1, and in it a chord $AB=p$, using any convenient unit of measure. Tangent to this chord and concentric with circle 1 draw circle 2. In circle 1 place a chord MNP such that $MP=r$ and $NP=s$. Through P draw chord CD tangent to circle 2. PC and PD are the roots of the quadratics. Their values may be found, using the same unit of measure as before.

Proof. $AB=CD=p$, $CP=x_1$, $PD=x_2=p-x_1$; $CP.PD=NP.PM=s.r$,

$$\left.\begin{array}{l} x_1(p-x_1)=rs,\quad x_1^2-px_1+rs=0 \\ \text{Similarly, } x_2^2-px_2+rs=0 \end{array}\right\}\text{equation 1.}$$

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*There are four cases, since a and b may each be positive or negative. The above solution suggests at once the following: Construct a circle of radius a; at the distance $\sqrt{b}$ from the diameter AB draw a parallel chord which intersects the circle (if at all) in $A'B'$. Draw $B'H$ perpendicular to AB; it is now evident that AH, HB are roots of the quadratic $(2a-x)x=b$; *i. e.* of $x^2-2ax+b=0$.

In order to solve $x^2-2ax=b$ we observe that by changing the signs of both roots the equation $x^2+2ax=b$ is obtained, and Mr. Holmes' solution applies.

Similarly, in order to solve $x^2+2ax+b=0$ we observe that its roots are the negatives of the roots of $x^2-2ax+b=0$, and the construction at the beginning of this foot-note applies. ED.

Equation 2, $x_2^2-px_2+q=0$, may be solved in a similar manner by changing the sign of px and proceeding as above. The roots being the same in numerical value but opposite in sign.

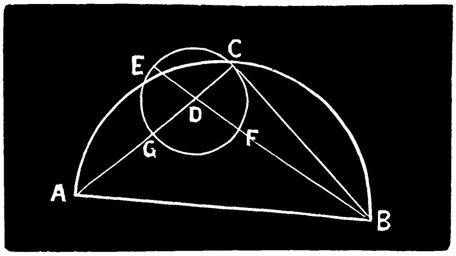
Case II. Equations 3, 4. The figure for Case II differs from that of Case I only in that P is outside of the circle 1; the points CPD being now in the order PCD. $PC=x_1$, $PD=x_2=x_1+p$, $PN.PM=PC.PD$; $r.s=x_1(x_1+p)$, $x_1^2+px_1-rs=0$. Similarly, $x_2^2+px_2-rs=0$.

Equation 4 is solved by changing sign of px and proceeding as with equation 3, since the roots are the opposite of the roots of equation (3).

In Case I if the roots are imaginary, the point P falls within circumference 2, and the graphic method fails.

VI. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Upon AB describe a semi-circle. Let C be the mid-point of the semi-circular arc. Draw AC, BC. Let G be a point on the line AC; on GC describe a circle center D. Through D draw $BFDE$. Then taking BE, BF positive; EB, FB negative, these lines are the roots of a quadratic having $2CD$ for the coefficient of the first power of the unknown quantity, and $\sqrt{(2AB)}$ for the absolute term, the coefficient of the second power of the unknown quantity being taken unity.



Let $AB=\sqrt{2c}$, $GC=2DC=b$.

$BE=BD+DE=BD+DC=\sqrt{(BC^2+DC^2)}+DC=\frac{1}{2}[b+\sqrt{(b^2+4c)}]$.

$BF=BD-DF=BD-DC=-\frac{1}{2}[b-\sqrt{(b^2+4c)}]$. Taking $x^2\pm bx=c$.

then for $x^2+bx=c$, $EB=-\frac{1}{2}[b+\sqrt{(b^2+4c)}]$, $BF=-\frac{1}{2}[b-\sqrt{(b^2+4c)}]$.

For $x^2-bx=c$, $BE=\frac{1}{2}[b+\sqrt{(b^2+4c)}]$, $FB=\frac{1}{2}[b-\sqrt{(b^2+4c)}]$.

If c be negative, the results still hold.

Also solved by G. W. Greenwood, B. A. (Oxon), Professor of Mathematics and Astronomy in McKendree College, Lebanon, Ill., by use of circle and hyperbola.

219A. Proposed by H. F. MacNEISH, A.B., Assistant in Mathematics. University High School, Chicago, Ill.

Draw a line through a given point which shall divide a given quadrilateral into two equivalent parts; (1) when the point lies in a side of the quadrilateral, (2) when the point is without, (3) within the quadrilateral.

Solution by G. B. M. ZERR. A. M., Ph. D., Parsons. W. Va.

Let $ABCD$ be the given quadrilateral. Produce DA, CB till they meet in F. Join AC and draw BS parallel to AC, join CS; then triangle SCD=quadri-$ABCD$. Disect SD in H, HF in G, and join CH, CG.

(1). Let P be the point in the side BC. Join PH and draw CL parallel to PH, join PL. Triangle PCL=triangle HCL.

$\therefore PCL+LCD=PCDL=HCL+LCD=HCD=\frac{1}{2}ABCD$.

(2). Let R be the point without the quadrilateral. Draw RK parallel to